# GRAND VALLEY
## STATE UNIVERSITY
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Analysis of the Pentad G motor from Gary Raymond Motors borrowed from Wyoming Rogers H. S.

The motor was set up on a test stand with a prony brake and data taken. The no-load data (labeled "1") is a measure of the fixed losses in the motor. The fixed loss is caused by the rolling torque of the bearings (bearing lubrication) and the magnetic hysteresis. As the voltage across the armature is increased, $V_A$, the armature current, $I_A$, and shaft RPM are recorded.

| $V_A$ | 6 | 8 | 10 | 12 | 14 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $I_{A1}$ | 3.105 | 3.893 | 4.02 | 4.08 | 4.146 | 4.147 | 4.118 | 4.132 | 4.173 | 4.179 | 4.241 | 4.351 | 4.362 | 4.463 |
| $RPM_1$ | 830.3 | 1119 | 1404 | 1687 | 1970 | 2256 | 2383 | 2515 | 2645 | 2777 | 2940 | 3077 | 3211 | 3352 |
| $I_{A2}$ | 10.0 | 10.584 | 10.39 | 10.362 | 10.63 | 11.463 | 11.47 | 11.85 | 12.568 | 13.927 | 16.116 | 18.23 | 19.607 | 9.281 |
| $RPM_2$ | 814 | 1120 | 1380 | 1668 | 1944 | 2245 | 2356 | 2487 | 2625 | 2746 | 2847 | 3027 | 3164 | 3333 |

TABLE #1, No-Load and Loaded Data

The no-load data should be linear so that a regression analysis will determine the intercept, b, and slope, m in the equation, $I_{A1} = m \cdot \omega_1 + b$. $\omega_1$ is the velocity of the motor shaft in radians per second and can be converted to RPM by, $RPM_1 = \omega_1 \cdot \frac{60}{2\pi}$. For the Pentad G motor these values are:

b = 3.3386(amp) and m = 0.0031581(amp-sec)

The intercept value, 3.3386(A), is the value of armature current that is required to just make the motor start to turn. This current occurs at an armature voltage of, $V_A = b \cdot R_A$ or $V_A = 0.1$(V). In other words, it takes a voltage of about 100(mV) across the armature to just start it turning without load.
With a load on the motor shaft, more current is required to produce the torque necessary to turn the shaft. In a DC motor, torque is proportional to the armature current. A constant for the motor, called the "motor constant" which includes the armature winding information as well as the magnetic circuit information, is defined as, $K\Phi$.

Torque is expressed as, $T(\text{newton-meter}) = K\Phi(\text{Nm/A}) \cdot I_A(\text{A})$

If the torque produced by the motor is larger than the torque load, the motor shaft will turn and accelerate. As the armature accelerates, a voltage is induced in the armature called a "counter EMF" which opposes the applied armature voltage and acts to reduce the armature current and therefore the torque produced. At some speed, the torque produced just equals the load torque and the counter EMF plus the voltage drop across the armature resistance equals the armature voltage. This condition is referred to as "steady-state". That is, the motor is not accelerating or decelerating but rather operating in an equilibrium condition where,

$$V_A = I_A \cdot R_A + K\Phi \cdot \omega.$$

At steady-state, the torque-speed curve for the motor can be derived from the equation above as,

$$\omega = \frac{V_A}{K\Phi} - \frac{R_A}{K\Phi^2} T = \frac{V_A - R_A I_A}{K\Phi}$$

The equation of the torque-speed curve for the Pentad G motor in terms of RPM, armature voltage and current is:

$$\mathrm{RPM} = 140.845 \cdot V_A - 4.225 \cdot I_A$$

For example, with an armature voltage of 23(volt) and armature current of 40(amp), the motor shaft RPM is: RPM = 3353.19 – 264 = 3089.19

Because the torque-speed curve for a DC motor is linear, the armature resistance, $R_A$, and the motor constant, KΦ, can both be calculated by loading the motor and recording the armature current and shaft speed in RPM at each value of $V_A$ and then averaged. The averaged values of $R_A$ and KΦ for the Pentad G motor are:

$$R_A = \frac{(\mathrm{RPM}_1 - \mathrm{RPM}_2)\cdot V_A}{(\mathrm{RPM}_1 \cdot I_{A2} - \mathrm{RPM}_2 \cdot I_{A1})}, \quad R_A = 0.03(\mathrm{ohms})$$

$$K\Phi = \frac{(I_{A2} - I_{A1})\cdot V_A}{(\mathrm{RPM}_1 \cdot I_{A2} - \mathrm{RPM}_2 \cdot I_{A1})\frac{\pi}{30}}, \quad K\Phi = 0.0678(\text{volt-sec}) \text{ or } (\mathrm{Nm/amp})$$

For a given armature voltage and current, the efficiency and power output of the motor can be found by:

$$\eta = 1 - \frac{R_A \cdot I_A}{V_A} - \frac{b}{I_A} + \frac{R_A \cdot b}{V_A} - \frac{V_A \cdot m}{K\Phi \cdot I_A} + \frac{2 \cdot R_A \cdot m}{K\Phi} - \frac{m \cdot R_A^2 \cdot I_A}{K\Phi \cdot V_A}$$

$$P_O = V_A \cdot I_A - R_A \cdot I_A^2 - V_A \cdot b + R_A \cdot b \cdot I_A - \frac{V_A^2 \cdot m}{K\Phi} + \frac{2 \cdot V_A \cdot R_A \cdot m \cdot I_A}{K\Phi} - \frac{R_A^2 \cdot m \cdot I_A^2}{K\Phi}$$

Using these equations for efficiency and power output, at an armature voltage of 23(volt) and an armature current of 40(amp), the efficiency and power output are: $\eta$ = 84.5% and $P_O$ = 777.1(watt) or 1.04(HP).
Each motor has a maximum efficiency and power output that is a function of load. In other words, for a given armature voltage, as the motor is loaded (armature current is increased), the motor produces power output and this power output is produced at a certain efficiency. As the motor is loaded, both power output and efficiency go through maximums. These maxima can be found at the following armature currents,

$$I_A(\max \eta) = \sqrt{\frac{\frac{V_A}{R_A}\left(b + \frac{V_A \cdot m}{K\Phi}\right)}{1 + \frac{m \cdot R_A}{K\Phi}}} \quad \text{and} \quad I_A(\max P_O) = \frac{\frac{V_A}{R_A} + b + \frac{2 \cdot V_A \cdot m}{K\Phi}}{2 \cdot \left(1 + \frac{R_A \cdot m}{K\Phi}\right)}.$$

For the Pentad G motor, the armature current for maximum efficiency occurs at $I_A$ = 58.1(amp), 2993.9(RPM) and for maximum power output, $I_A$ = 385.5(amp), 1610.4(RPM). At these currents (load) the maximum efficiency is $\eta_{max}$ = 85.5% and $P_{Omax}$ = 4363.9(watt) or 5.9(HP).